



Begin by circumscribing a circle around the triangle in question, ABC. (A unique circle can be drawn through any three points). Construct diameter AOD and join CD. Let the radius be r .

ΔACD is a triangle in a semicircle and therefore $\angle ACD = 90^\circ$.

In right angled ΔACD ,

$$\begin{aligned} \sin D &= AC/AD \\ \text{i.e. } \sin D &= AC/2r \dots\dots (i) \end{aligned}$$

As angle $B = \text{angle } D$ (because they stand on same arc AC), (i) above may be written

$$\begin{aligned} \sin B &= AC/2r \\ \text{so } 2r &= AC/\sin B \\ \text{i.e. } \mathbf{2r} &= \mathbf{b/\sin B} \end{aligned}$$

In a similar way, (drawing diameter from C etc) it can be shown that

$$\begin{aligned} 2r &= BC/\sin A \\ \text{i.e. } \mathbf{2r} &= \mathbf{a/\sin A} \end{aligned}$$

and

$$\begin{aligned} 2r &= AB/\sin C \\ \text{i.e. } \mathbf{2r} &= \mathbf{c/\sin C} \end{aligned}$$

We now have three expressions for $2r$, therefore

$$a/\sin A = b/\sin B = c/\sin C$$